The Radiometer Equation
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(www.youtube.com/watch?v=dwT-tMoscsY & casper.berkeley.edu/astrobaki/index.php/Radiometer_Equation_Applied_to_Telescopes)

The radiometer equation, for a bandwidth of $\Delta\nu$ (MHz) and integration time, τ is

$$\frac{S}{N} = \frac{T_{src}}{T_{sys}}\sqrt{\Delta\nu\tau}. \tag{1}$$

Deriving the Radiometer Equation

Radio astronomers use temperatures because they began as engineers who think about resistors that heat up when you put power through tem. Change the temperature to fluxes using the Rayleigh-Jeans expression for specific intensity:

$$I_\nu = \frac{2kT_{src}}{\lambda^2} \sim \frac{erg}{s \bullet cm^2 \bullet Hz \bullet str}. \tag{2}$$

The flux density is just the intensity integrated over the solid angle of the sky seen by the antenna, Ω_a,

$$S_\nu = \int I_\nu d\Omega = \int \frac{2kT_{src}}{\lambda^2}\, d\Omega \sim \frac{erg}{s \bullet cm^2 \bullet Hz} \tag{3}$$

Where 1 Jy = 10^{-23} erg/s•cm^2•Hz. The power received by the antenna is the intensity integrated over the effective area of the antenna, A_e, and the solid angle of the sky seen by the antenna, Ω_a.

$$P_\nu = \int S_\nu dA = \iint I_\nu d\Omega dA$$

The antenna theorem states that the effective area of the antenna, times the solid angle it subtends is equal to the observed wavelength squared.

$$A_e \Omega_a = \lambda^2 \tag{4}$$

Thus,

$$P_\nu = I_\nu \iint d\Omega dA$$

$$P_\nu = \frac{2kT_{src}}{\lambda^2}\left(\lambda^2\right)$$

$$P_\nu = 2kT_{src} \tag{5}$$

To derive the conversion from the source temperature to the flux density of an unresolved source, write the power in terms of the flux density,

$$\begin{aligned} P_v &= S_\nu A_e \\ 2kT_{src} &= S_\nu A_e \\ T_{src} &= \left(\frac{A_e}{2k}\right) S_\nu \end{aligned} \tag{6}$$

Where the factor, $A_e/2k$ is known as the forward gain of the antenna in K/Jy or Jy/K.

The effective area, in terms of the peak antenna gain is defined as[1]

$$A_e = \frac{\lambda^2}{4\pi} G \tag{7}$$

Back to the signal-to-noise, The system temperature is a combination of the sky (eg. noise generated above antenna that we don't want to detect such as our own galaxy, nearby sources, CBR, etc.) temperature and the temperature of the receiver, the thermal (or Johnson) noise of the electrical components of the receiver:

$$T_{sys} = T_{sky} + T_{receiver}.$$

And, typically $T_{src} < T_{sys}$. To detect objects, we need to beat down the noise which is defined as T_{rms} and is given by

$$T_{rms} = \frac{T_{sys}}{\sqrt{n}}$$

where n is the number of data points, equal to the bandwidth times the integration time:

$$T_{rms} = \frac{T_{sys}}{\sqrt{n}} = \frac{T_{sys}}{\sqrt{\Delta\nu\tau}}$$

The signal-to-noise ratio, then is

$$\frac{S}{N} = \frac{T_{src}}{T_{rms}} = \frac{T_{src}}{\left(\frac{T_{sys}}{\sqrt{\Delta\nu\tau}}\right)} = \frac{T_{src}}{T_{sys}}\sqrt{\Delta\nu\tau}$$

which is what we set out to derive!

[1] Antennna-Theory.com (www.antenna-theory.com)

RADIOMETER EQUATION IN TERMS OF FLUX DENSITY

Define the System Equivalent Flux Density (SEFD ~ Jy) as the flux density equivalent of T_{sys}, found by rearranging (6) to

$$T_{src} = \left(\frac{A_e}{2k}\right) S_\nu \quad \Rightarrow \quad S_{sys,rms} = \frac{2kT_{sys}}{A_e} = SEFD$$

and write the rms flux density variations (instead of rms system temperature variations) is then

$$S_{\nu,rms} = \frac{SEFD}{\sqrt{\Delta\nu\tau}}$$

showing that increasing the integration time decreases the flux density of the noise. The parameters used in these equations are available for the Arecibo receivers in *An Astronomer's Guide to the Arecibo 305-m Telescope* by Chris Salter at www.naic.edu/~astro/guide.

Receiver Designation	Freq Range (GHz)	System Temp[a] (K)	Gain[a] K/Jy	SEFD[a,b] Jy (at zenith)	HPBW[a,f] Az × ZA (Arcmin)
Carriage House					
47	-	T_{sky}	-	-	94 × 110
430ch	0.425 − 0.435	70 - 120	10 - 20	3.5 - 10	10 × 12
Gregorian Dome: Single-Pixel Receivers					
327	0.312 − 0.342	90 + T_{sky}	10.5	11	14 × 15
430	0.425 − 0.435	35 + T_{sky}	11	5	10 × 12
800	0.705 − 0.800	110	9.5	12	~6 × 7
lbw	1.120 − 1.730	25	10.5	2.4	3.1 × 3.5
sbw	1.800 − 3.100	32	9.5	3.4	1.8 × 2.0
sb	2.240 − 2.340	25	10	2.5	1.8 × 2.0
	2.330 − 2.430	25	10	2.5	1.8 × 2.0
sbh	3.000 − 4.000	29	8.8	3.3	1.35 × 1.5
cb	3.850 − 6.050	31	8	3.9[c]	0.9 × 1.0
cbh	6.000 − 8.000	28	5.5	5[d]	0.65 × 0.75
xb	8.0 − 10.0	33	4.5	7.5[e]	0.5 × 0.6
Gregorian Dome: Feed Array					
ALFA					
Center Pix	1.225 − 1.525	30	11	2.8	3.3 × 3.7
Outer Pixs	1.225 − 1.525	30	8.5	3.5	3.3 × 3.7

Receiver Notes

a) T_{sys}, Gain and SEFD all vary with zenith angle (and to a lesser degree with azimuth). T_{sys} and SEFD increase with zenith angle, while Gain decreases. The HPBW in ZA increases with zenith angle.

b) SEFD, the System Equivalent Flux Density (= T_{sys}/G) is the system temperature expressed in Jy/beam.

c) At 5 GHz.

d) At 6.9 GHz.

e) At 9 GHz.

f) HPBW is the Half-Power Beam-Width.